European Drag Reduction and Flow Control Meeting – EDRFCM 2015
March 23–26, 2015, Cambridge, UK

RELAMINARISATION OF TURBULENT CHANNEL FLOW WITH PASSIVITY-BASED LINEAR FEEDBACK CONTROL

K. Septham, J. F. Morrison
Department of Aeronautics, Imperial College London, SW7 2AZ, London, UK

ABSTRACT

Full-domain, linear feedback control of fully-developed turbulent channel flow at $Re_\tau \leq 400$ is an effective method to attenuate turbulent channel flow such that it is relaminarised (figure 1). The passivity-based control approach is adopted and explained by the conservative characteristics of the non-linear terms in the Navier-Stokes equations with respect to the disturbance energy [5]. The implementation and testing of a control algorithm are performed in a plane Poiseuille channel flow using a direct numerical simulation (DNS). A modified version of Channelflow-1.4.2 [2], available under the GNU General Public License, is employed.

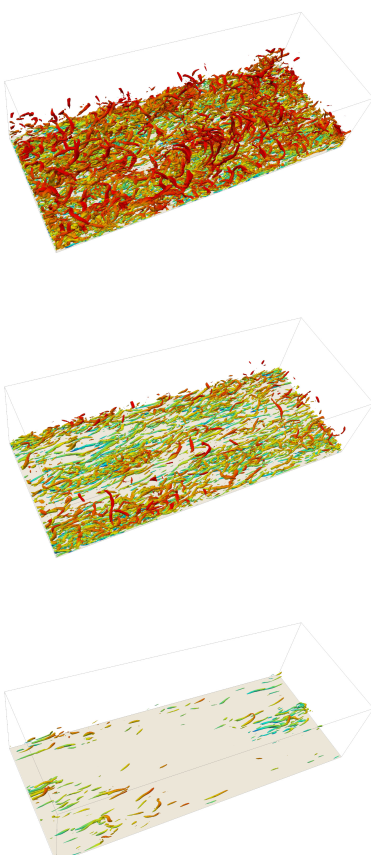

Figure 1: Instantaneous isosurfaces of λ_2 in relaminarising turbulent channel flow at $Re_\tau = 400$: $t = 0$, $t = 10$, and $t = 40$, respectively.

Firstly, the linear feedback control operating on the wall-normal velocity fluctuation is restricted to low wavenumbers which correspond to the large energy-containing eddies, while the significant viscous effects are sufficient to dissipate energy at the highest wavenumbers. Then, the restriction of control is progressively increased to higher wavenumbers until the mean pressure gradient uninterruptedly decreases (figure 2).

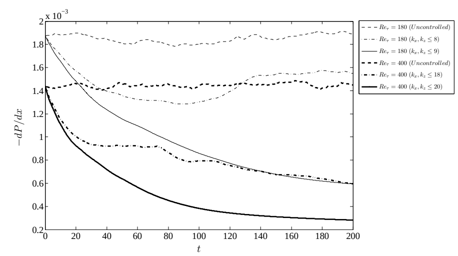

Figure 2: Temporal evolution of the mean pressure gradient for the uncontrolled and controlled flows at $Re_\tau = 180$ and 400.

Figure 2 shows that the minimum restricted streamwise and spanwise wavenumbers without losing control $(k_x, k_z)_{min}$ increase with increasing Re_τ, as would be expected from a broader range of scales at higher Reynolds numbers. The minimum required spanwise wavelength resolution without losing control

$$\lambda^+_{z,min} = \left(\frac{2\pi}{k_{z,min}}\right)\left(\frac{u_\tau}{\nu}\right) \tag{1}$$

is calculated at each friction Reynolds number, where the value of the friction velocity u_τ at time $t = 0$ is used. It reveals that $\lambda^+_{z,min}$ is approximately equal to 125 and it is invariant with the friction Reynolds number (figure 3).

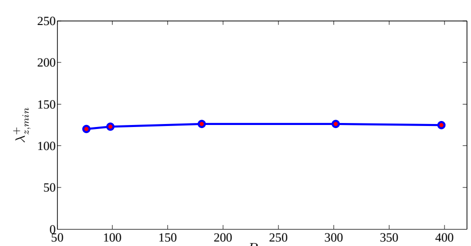

Figure 3: The minimum required spanwise wavelength resolution without losing control for the controlled flows at $Re_\tau = 80, 100, 180, 300$, and 400.

Furthermore, it shows that in the short term the forcing peak is located at $y^+ \approx 25$ (figure 4) which corresponds to the location of the maximum mean-square pressure gradient of the fully-developed unmanipulated turbulent channel flow (figure 5). This could be understood via the mean-square acceleration equation [1]:

$$\overline{\left(\frac{Du_i}{Dt}\right)^2} = \overline{\left(\frac{\partial p}{\partial x_i}\right)^2} + \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2} - 2\nu \frac{\partial}{\partial x_i}\left(\overline{p\frac{\partial^2 u_i}{\partial x_i^2}}\right). \quad (2)$$

Batchelor and Townsend (1956) have shown that the viscous diffusion term is negligible at sufficiently high Reynolds numbers where the hypothesis of local isotropy is valid. Thus, the mean-square acceleration of a fluid particle at high Reynolds numbers is driven by the mean-square pressure gradient and the mean-square viscous force,

$$\overline{\left(\frac{Du_i}{Dt}\right)^2} \approx \overline{\left(\frac{\partial p}{\partial x_i}\right)^2} + \nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}. \quad (3)$$

They have further suggested that the mean-square pressure gradient is significantly larger than the mean-square viscous force and

$$\overline{\left(\frac{\partial p}{\partial x_i}\right)^2} \approx 20\nu^2 \overline{\left(\frac{\partial^2 u_i}{\partial x_j^2}\right)^2}. \quad (4)$$

Consequently, it is likely that the controller works against the mean-square pressure gradient over time and the near-wall motion is driven by prolonged viscous periods periodically pulsed by the pressure fluctuations.

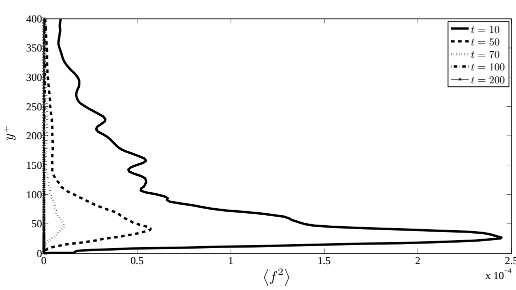

Figure 4: Mean-square forcing $\left\langle f^2 \right\rangle(y,t)$ of the linear control restricted to $(k_x, k_z) \leq 20$ for the controlled flow at $Re_\tau = 400$.

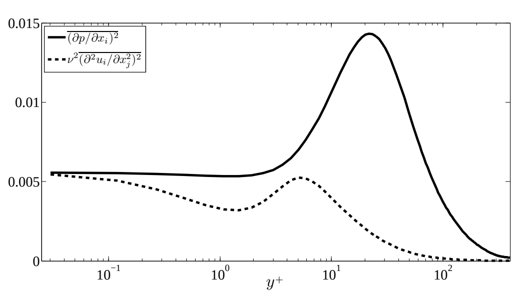

Figure 5: Mean-square pressure gradient and mean-square viscous force of fully-developed turbulent channel flow at $Re_\tau = 400$, according to equation 3.

Further investigation on the rate of change of the total perturbation energy dE/dt over a closed domain Ω is carried out using the Reynolds-Orr equation, given by

$$\frac{dE}{dt} = -\oint_\Omega \left(uvU_L'(y) + \frac{1}{Re}\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}\right) d\Omega = P_E + D_E. \quad (5)$$

Clearly, the linear control operates via vU' ($U' = \partial U/\partial y$) (figure 6). The effectiveness of the linear control is qualitatively explained by Landahl's theory for timescales [4], in that the control proceeds via the shear interaction timescale which is significantly shorter than both the nonlinear and viscous timescales for the turbulence. Over a short period of time for which the linear control is in effect, the longer nonlinear (turbulence) timescale is not significant.

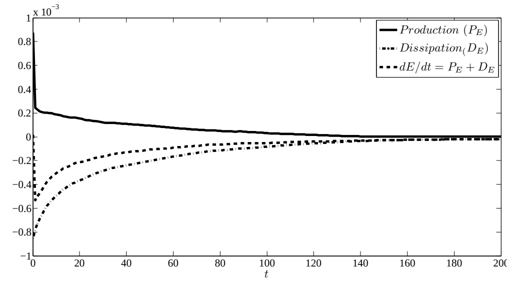

Figure 6: The temporal evolution of the rate of production P_E and the dissipation rate D_E of the perturbation energy for turbulent channel flow at $Re_\tau = 400$ subject to the linear control restricted to $(k_x, k_z) \leq 20$

Lastly, the response of the rapid (linear) and slow (nonlinear) pressure fluctuations to the linear control are investgated using the Green's function representations [3]. It demonstrates that the linear control operates via the rapid (linear) source term of the Poisson equation for pressure fluctuations, $2U'\frac{\partial v}{\partial x}$ and the shear interaction timescale is effective as a result of the rapid source term of the Poisson equation for pressure fluctuations.

REFERENCES

[1] G. K. Batchelor and A. A. Townsend. *Surveys in Mechanics*, chapter Turbulent diffusion, pages 352–399. Cambridge University Press, Cambridge, UK, 1956.

[2] J. F. Gibson, J. Halcrow, and P. Cvitanović. Visualizing the geometry of state space in plane Couette flow. *J. Fluid Mech.*, 611:107–130, 2008. `arXiv:0705.3957`.

[3] J. Kim. On the structure of pressure fluctuations in simulated turbulent channel flow. *Journal of Fluid Mechanics*, 205:421–451, 8 1989.

[4] M. T. Landahl. Model for the wall-layer structure of a turbulent shear flow. *Eur. J. Mech. B/Fluids*, 12:85–96, 1993.

[5] A. S. Sharma, J. F. Morrison, B. J. McKeon, D. J. N. Limebeer, W. H. Koberg, and S. J. Sherwin. Relaminarisation of $Re_\tau = 100$ channel flow with globally stabilising linear feedback control. *Physics of Fluids*, 23(125105), 2011.