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WAVE PROPERTIES OF A COMPLIANT COATING IN A FLUID FLOW

Vasily Vedeneev
Department of Hydromechanics, Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow, Russia

INTRODUCTION

Reduction of drag force is a challenging hydrodynamic problem for bodies moving in a fluid. One of perspective methods of passive flow control is the compliance of the surface. A series of papers is devoted to the investigation of laminar boundary layer stability over compliant surfaces [1, 2, 3, 4, 5, 6, 7], where the efficiency of the compliance for the laminar-turbulent transition delay is established.

Interaction of compliant coating with turbulent boundary layers is less studied. Two approaches have been proposed. First, a few papers are devoted to direct numerical simulation of the fluid-structure interaction [8, 9]. The second, semi-empirical approach consists of a series of criteria [10] that the coating should satisfy to be optimal for drag reduction. In contrast to DNS, the effeciency of the second approach was proved by experiments [11, 12]. Criteria [10] use such coating parameters as thickness, stiffness, phase speed of surface waves, etc., i.e. the parameters that are easily measured or calculated.

Recently, it was shown [13] that for coatings used in tests [11, 12], representation as a viscoelastic continuum is much more accurate than as a spring-supported plate assumed in most theoretical studies. In particular, phase speeds of travelling waves and resonant effects are well described by this model. However, in case of coatings operating in water, their wave properties differ from those measured in lab due to influence of the inertia of water. In this paper we derive the dispersion relation for a layer of viscoelastic coating in a water layer in order to estimate influence of the water flow on the wave properties of the coating.

FORMULATION OF THE PROBLEM

We study linear waves in a layer of a linear viscoelastic material of the thickness H. On side of the layer is rigidly fixed, while the other is in contact with a layer of inviscid incompressible fluid of the thickness L (fig. 1). The fluid moves with a constant velocity u, which is parallel to the layer. The problem is considered in 2-D formulation.

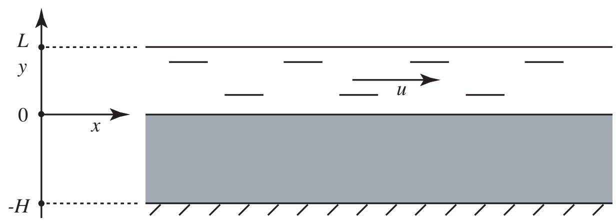

Figure 1: Layer of incompressible fluid over the layer of viscoelastic material.

Since we study oscillatory motions of the system, we use the concept of complex dynamic moduli for modeling viscoelastic properties. Namely, the constitutive equations (Hooke's law) and the governing equations are the same as for linear elastic medium, but elastic moduli representing material properties are generally complex. Their real parts represent elastic behaviour, while imaginary parts represent dissipation of energy in oscillatory motion.

DISPERSION RELATION

Introduce the coordinate system as shown in fig. 1. Let $\{w_x, w_y\}$ be the displacement vector of elastic medium, Φ the perturbation of the potential of the fluid. Consider motion of the system in form of a travelling wave:

$$w_x = f(y)e^{ik(x-ct)}, \quad w_y = g(y)e^{ik(x-ct)}, \quad \Phi = s(y)e^{ik(x-ct)}.$$

Using equations of linear elasticity for the medium and Laplace equation for the fluid motion $\Delta\Phi = 0$, connecting the medium and the fluid through the boundary conditions along the interface, we derive the dispersion relation:

$$2\left[\left(2-\left(\frac{c}{a_2}\right)^2\right)^2\left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \frac{\sinh(\lambda_1 H)\sinh(\lambda_2 H)}{\alpha\beta}\right) + \right.$$
$$\left. + \, 4\left(\cosh(\lambda_1 H)\cosh(\lambda_2 H) - \alpha\beta\sinh(\lambda_1 H)\sinh(\lambda_2 H)\right) - 4\left(2-\left(\frac{c}{a_2}\right)^2\right)\right] =$$
$$= m\left(\frac{c}{a_2}\right)^2\left(\frac{u-c}{a_2}\right)^2\frac{1}{\beta}\tanh(kL)\left[(1-\alpha\beta)\sinh((\lambda_1+\lambda_2)H) + (1+\alpha\beta)\sinh((-\lambda_1+\lambda_2)H)\right] \quad (1)$$

Here a_1 and a_2 are the speeds of longitudinal and transversal waves in the viscoelastic medium, m is the fluid density rated to the material density of the coating. It can be shown that the influence of gravity is essential only for waves of lengths $\lambda \sim 100H$ or more. Since we do not consider so large lengths, the gravity terms are neglected in (1). Note that neglecting the right-hand side of the equation (1) yields the dispersion relation for viscoelastic layer in vacuum [13].

SURFACE INSTABILITY

Investigation shows that the water flow can yield instability of the system. Shown in fig. 2 are numerically obtained first two roots $c(u)$ of the dispersion relation (1) for a certain value of k. For $u = 0$, i.e. fluid at rest, phase speeds of downstream and upstream travelling waves are equal but lower than in the case of the absence of fluid. When the flow speed increases, downstream travelling wave moves faster, while the upstream travelling wave moves slower and converts to downstream travelling. For higher u two phase speeds approach each other and coalesce at $u = u_{cr}$. For $u > u_{cr}$, real roots are substituted by complex-conjugate roots resulting in instability.

We have conducted asymptotic analysis for short and long waves ($k \to \infty$ and $k \to 0$, respectively) and numerical investigation for arbitrary k. It is shown that the lowest u_{cr} is achieved for $k \to \infty$. Stability boundary in the parameter space is obtained.

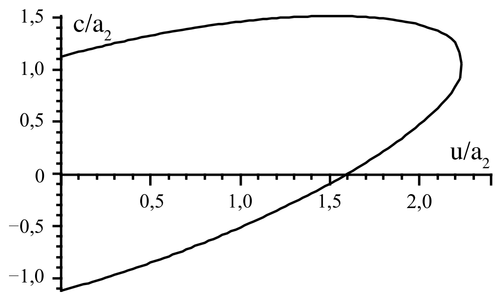

Figure 2: The first two roots of the dispersion relation $c(u)$, $m = 1$.

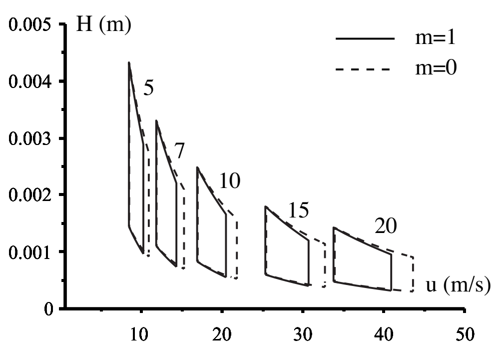

Figure 3: Regions of the u–H plane, where the criteria of the compliant coating effectiveness are satisfied, for different values of the transversal wave speed a_2 (m/s).

MODIFICATION OF THE CRITERIA FOR COMPLIANT COATING EFFICIENCY

Modification (1) of the dispersion relation [13] (namely, taking the fluid into account) yields two corrections of optimal compliant coating criteria. First, according to [13], maximum interaction of the coating with the flow is achieved for $c = (0.7...0.9)u$. Since the root c for the wave in fluid is less than for the wave in vacuum, we conclude that optimal flow speed for a given coating is slightly lower in the fluid. That is, the maximum effect of the coating in water is achieved at lower speeds than in air. As an example, regions satisfying compliant coating effeciency criteria in water ($m = 1$) and in air ($m = 0$) are shown in fig. 3.

Second, the surface instability must be avoided for the coating to be efficient. This yields additional restriction on the material density of the coating.

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