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TRANSITION DELAY IN PLANE CHANNEL FLOW USING SUPERHYDROPHOBIC COATING

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INTRODUCTION

Superhydrophobic coatings are becoming an increasingly popular technique for the reduction of drag in applications involving the flow of liquids over solid surfaces, for a wide range of Reynolds number, from laminar to turbulent conditions. Such coatings work by the interposition of a gas layer between the liquid and the solid wall, trapped by distributed microscopic roughness elements present at the wall; over the gas layer the liquid can flow with negligible friction. Here we are concerned with the initial development stages of the laminar-turbulent transition for the flow of a liquid in a micro-channel, with one or both walls characterized by a periodic, micro-patterned topography. The surface topography is rendered in the equations by a Navier-slip condition [3, 1], which mimics the alternating no-slip/no-shear boundary condition which applies when the air-liquid interface is underformed (i.e. infinite surface tension). Linear stability results are presented for both modal and nonmodal amplification of disturbances, for micro-ridges aligned, orthogonal, or at an angle, to the driving pressure gradient. Finally, a new weakly nonlinear approach [4] is used to find threshold amplitudes for the onset of transition.

PROBLEM FORMULATION

Ridges form an anisotropic texture, and a slip tensor $\mathbf{\Lambda}$ in the plane of the walls (x, z) can be defined as

$$\mathbf{\Lambda} = \mathbf{Q}\begin{bmatrix}\lambda^{\parallel} & 0\\ 0 & \lambda^{\perp}\end{bmatrix}\mathbf{Q}^T, \quad \text{with} \quad \mathbf{Q} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} \tag{1}$$

where $\lambda^{\parallel}$ and $\lambda^{\perp}$ are the Navier slip lengths along and orthogonal to the mean flow. In the special case of an isotropic superhydrophobic surface $\lambda^{\parallel} = \lambda^{\perp}$; for the case of micro-ridges it is $\lambda^{\parallel} = 2\lambda^{\perp}$ [3, 1]. By allowing for θ (angle between the ridges and the x-axis) to be different from zero, we can align the microstructures at any angle with respect to the mean pressure gradient. We assume that the channel has thickness $2l$ and use l to normalize distances; the bulk speed is employed to scale the velocity. The dimensionless boundary conditions at the two walls read

$$\begin{bmatrix}u(x,-1,z)\\ w(x,-1,z)\end{bmatrix} = \mathbf{\Lambda}\frac{\partial}{\partial y}\begin{bmatrix}u(x,-1,z)\\ w(x,-1,z)\end{bmatrix}, \tag{2}$$

$$\begin{bmatrix}u(x,1,z)\\ w(x,1,z)\end{bmatrix} = -\mathbf{\Lambda}\frac{\partial}{\partial y}\begin{bmatrix}u(x,1,z)\\ w(x,1,z)\end{bmatrix}, \tag{3}$$

in the case of both walls being textured, plus vanishing conditions for the vertical velocity component v at the two walls. If one of the two walls is not superhydrophobic, the condition there is simply $\mathbf{u} = \mathbf{0}$.

It is interesting to observe that, when θ differs from 0° and 90°, a small component of the base flow orthogonal to the mean pressure gradient is created in the channel [5]; when both walls are superhydrophobic the base flow is

$$U(y) = -3\frac{y^2 - [\lambda^{\parallel}(1+\cos^2\theta)+1]}{2+3[\lambda^{\parallel}(1+\cos^2\theta)]}, \tag{4}$$

$$W(y) = 3\frac{\lambda^{\parallel}\sin\theta\cos\theta}{2+3[\lambda^{\parallel}(1+\cos^2\theta)]}. \tag{5}$$

An example of base flow for $\lambda^{\parallel} = 0.1$ is displayed in figure 1 for two values of θ; the spanwise component W is constant and positive for $\theta = 45^o$.

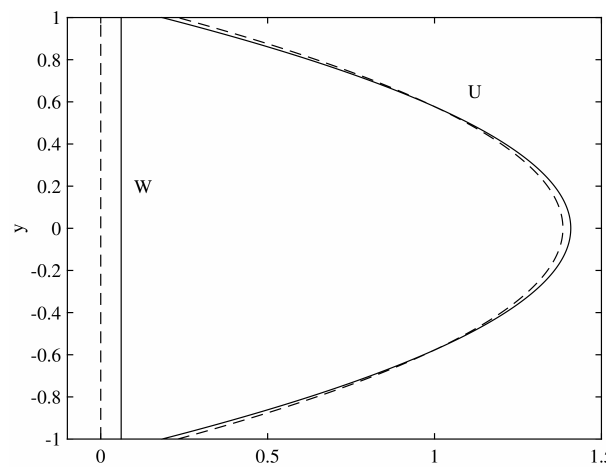

Figure 1: Streamwise U and spanwise W velocity components of the base flow when $\lambda^{\parallel} = 0.1$ for the cases $\theta = 0°$ (dashed) and $\theta = 45°$ (solid).

Linear stability equations are derived by introducing into the Navier-Stokes equations a flow decomposition $\mathbf{u}(x, y, z, t) = (U, 0, W)(y) + \epsilon\tilde{\mathbf{u}}(y, t)\exp[i(\alpha\, x + \beta\, z)] + \text{c.c.}$, where α and β are the streamwise and spanwise wavenumbers, respectively, and collecting terms of order ϵ. The theory is applicable as long as the disturbance wavelength is sufficiently longer than the spatial periodicity of the ridges.

A modal analysis is performed by assuming a temporal behaviour such that $\tilde{\mathbf{u}}(y, t) = \hat{\mathbf{u}}(y)\exp(-i\,\omega\, t)$, where ω is the complex angular frequency and $\omega_i > 0$ denotes unstable solutions.

The non-modal behaviour is studied by computing the maximum finite-time amplification; the initial disturbance velocity field, $\tilde{\mathbf{u}}_0$, is *optimal* when the gain

$$G(Re, \alpha, \beta, T, \lambda^{\parallel}, \theta) = \frac{e(T)}{e(0)}, \tag{6}$$

is maximized, where

$$e(t) = \frac{1}{2}\int_{-1}^{1} (\tilde{u}\tilde{u}^* + \tilde{v}\tilde{v}^* + \tilde{w}\tilde{w}^*) dy,$$

and T is the target time of the optimization. This is conducted by introducing Lagrange multipliers enforcing the constraints given by the governing linear equations and the boundary conditions. The adjoint equations are derived using a discrete approach. We further define

$$G_M(Re, \lambda^{\parallel}, \theta) = \max_{\forall \alpha, \beta, T} G,$$

when G is maximized with respect to wavenumbers (α, β) and the final time T.

RESULTS

The onset of the instability is studied parametrically by varying the parameters $Re, \alpha, \beta, T, \lambda^{\parallel}$ and θ. Results of the critical Reynolds number as a function of the wave angle, $\Phi = \tan^{-1}(\beta/\alpha)$, from the modal analysis are presented in figure 2 for two different superhydrophobic coatings on both walls of the channel. A monotonic increase of the critical Reynolds number as a function of Φ is found when the micro-ridges are orthogonal to the mean pressure gradient, showing that the two-dimensional wave is the least stable one, in accordance with Squire's theorem. When $\theta = 0$, on the other hand, the least stable disturbance, for the given value of $\lambda^{\parallel}$, is three-dimensional. By way of comparison, the critical Reynolds number in the no-slip case is 3848, demonstrating the stabilizing effect of the superhydrophobic walls for exponentially growing disturbances, particularly when the micro-ridges are aligned with the basic pressure gradient.

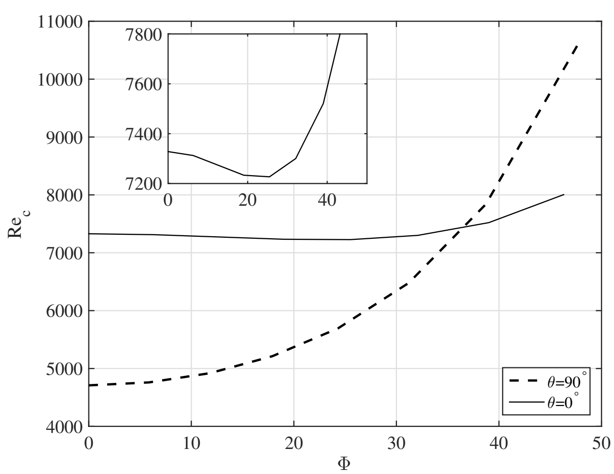

Figure 2: Critical Reynolds number as a function of the wave angle $\Phi = \tan^{-1}(\beta/\alpha)$ for $\lambda^{\parallel} = 0.02$.

The gain G_M obtained from the non-modal analysis is displayed as function of $\lambda^{\parallel}$ in figure 3 for $Re = 1260$ and $\theta = 0$. Also in this case we consider two identical superhydrophobic bounding surfaces. The results show that there is a monotonic decrease of the finite-time amplification as the Navier slip length $\lambda^{\parallel}$ rises. In the case of no-slip walls, the maximum gain is 700; it is thus convenient to enhance $\lambda^{\parallel}$ as much as possible to reduce transient amplification.

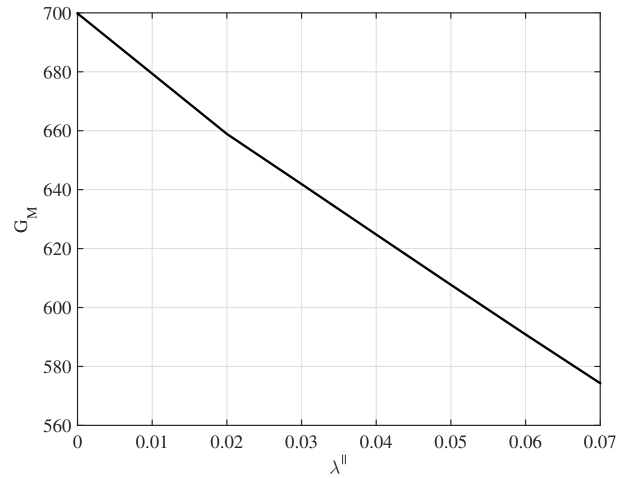

Figure 3: Gain G_M as a function of $\lambda^{\parallel}$ in the case of $\theta = 0$ and $Re = 1260$.

It has been known for some time that modal and nonmodal linear mechanisms are weak indicators of transition to turbulence in plane channel flow. We have thus considered a weakly nonlinear optimal model capable to describe the feedback occurring between the mean flow and the disturbances, along the lines of Pralits & Bottaro [4]. Weakly nonlinear optimal disturbances display a shorter streamwise and a longer spanwise wavelength than their linear counterparts, over a range of Reynolds numbers. Threshold values of the initial excitation energy, separating the region of damped waves from that where disturbances grow without bounds, scale like Re^{-2}.

CONCLUSIONS

A linear and weakly nonlinear analysis of the flow in a channel with the walls coated with a superhydrophobic material has been conducted, for the case of surface topography constituted by micro-ridges with arbitrary alignment. The results of the linear study are in agreement to and complete those by Min & Kim [2]. Nonlinear results permit, for the first time, to identify threshold amplitudes of disturbances provoking transition for the flow in a micro-channel bound by one or two superhydrophobic surfaces.

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