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THE RELEVANCE OF LONGITUDINAL AND TRANSVERSE PROTRUSION HEIGHTS FOR DRAG REDUCTION BY A SUPERHYDROPHOBIC SURFACE

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INTRODUCTION

When riblets were being extensively studied as drag-reduction devices several years ago, the protrusion height was introduced [1] as an important parameter quantifying their effectiveness. The underlying idea is that riblets are small enough that flow in their neighbourhood is governed by the Stokes equations, even when the surrounding flow is turbulent, and therefore their action is linear and can be replaced by the asymptotic solution of the Stokes equations themselves; this is a Couette velocity profile originating at a virtual wall located at a certain height, which takes the name of protrusion height. More precisely, as established in [6], there are two protrusion heights, one for flow along and one for flow across the riblets, and since the reference plane from which these heights are measured is arbitrary and has no physical meaning, only the protrusion-height *difference* is the relevant physical parameter. The whole idea of protrusion height reappears in the form of the slip length that is today used to describe the drag-reducing action of superhydrophobic surfaces; however the distinction between a longitudinal and a transverse protrusion height and the importance of their difference does not seem to have been realized yet in this context.

THE RELATIONSHIP BETWEEN PROTRUSION-HEIGHT DIFFERENCE AND DRAG REDUCTION

In a turbulent flow the profile of mean velocity, which only has a longitudinal component if the riblets are aligned with the main flow, will appear to originate at the longitudinal protrusion height. In order to obtain a crude quantitative relationship between the numerical value of the protrusion-height difference and the ensuing drag reduction, the present author in [5] made a somewhat drastic assumption: that the turbulent fluctuations see their effective wall at the transverse protrusion height. This may be conceptually justified if one believes longitudinal vortices (having a transverse velocity near the wall) to play a significant role in the self-sustaining process of such fluctuations.

Once this assumption is made, it follows that the turbulent mean-velocity profile over a wall with riblets will be described by the same law-of-the-wall as over a plane wall, except for a displacement by an amount equal to the difference between the two protrusion heights. Denoting this difference by Δh, the relative drag reduction was shown in [5] to be given by

$$\frac{\Delta c_f}{c_f} = -\frac{\Delta h^+}{(2c_f)^{-1/2} + (2\kappa)^{-1}} \tag{1}$$

where c_f is the friction coefficient, κ Von Kàrmàn's constant, and Δh^+ is Δh normalized in wall units.

It can be noticed that, according to this formula, Δc_f is directly proportional to Δh. It cannot be otherwise in a linearized model. Δh^+ is in turn a function of s^+, the dimensionless spacing (repetition period) of the riblets, as exemplified for a number of shapes in [6]. Experimentally, the drag coefficient of typical riblets can be seen to attain a minimum for $s^+ \simeq 15$, then to increase again to its flat-wall value for $s^+ \simeq 30$ and well beyond thereafter. The initial, decreasing, part of the experimental curve, although somewhat difficult to be measured, is in reasonable agreement with (1) [2]. Recent numerical simulations also confirm this agreement [3].

SUPERHYDROPHOBIC SURFACES

There is nothing in the above reasoning that exclusively applies to riblets. Any wall modification that occurs on a small enough scale for its action to be limited to the Stokes sublayer must follow the same principles. Superhydrophobic surfaces change the macroscopic contact angle of liquid drops because they are micro-structured so as to embed microscopic air bubbles. While contact angle per-se has no relevance once the wall is completely covered with a flowing liquid, the presence of the air pockets creates a slip velocity that eventually changes the drag coefficient. For this reason they are being considered for drag reduction in turbulent flow, more sensitive to this slip than laminar flow for its higher velocity gradient at the wall. The stability of the air pockets against being carried away by the stream is a delicate issue, which will not be examined here other than by saying that it places an upper limit on their size for surface tension to be the dominant force.

Very small air pockets will again only influence the Stokes sublayer of the turbulent flow above. Their action on a Stokes flow and the corresponding slip length (protrusion height) was studied in [4], without however making an explicit connection to turbulent flow. In fact, the authors of [4] and many others made an additional simplifying assumption: that surface tension is so large (the air pockets are so small) that the air-water interface is virtually flat. The superhydrophobic surface can then be modelled as an overall flat surface with alternating patches of no-slip and no-shear boundary condition. It turns out that such a patchwork plane surface had already been studied (for other purposes) by Philip [8] who proposed several analytic solutions for the Stokes flow, including the case of both longitudinal and transverse flow past a striped surface, one where the air pockets (and the intervening solid wall) take the form of infinitely long stripes.

DIRECT NUMERICAL SIMULATION

Superhydrophobic surfaces are a subject of intense research these days, and not all references can be listed here. Just as an example, a recent direct numerical simulation of turbulent flow past a superhydrophobic surface [7] was accompanied by an extensive correlation between the drag reduction obtained in the simulation and the (longitudinal) slip length, for the case of a striped no-shear, no-slip surface like the one considered in [8]. However, these authors used a range of relatively large values of s^+ (P^+ in their figure 8), and did not really explore the neighbourhood of $s^+ \simeq 15$ where riblets work. In fact exploring such region poses an additional numerical difficulty, because the large scale separation between the spacing of the stripes and the computational periodic box may force the use of a very fine numerical discretization.

In our own direct numerical simulation, which will be presented here, this difficulty has been overcome through a local analytical correction to the numerical discretization. At the same time this correction takes care of the square-root singularity that exists at the edge between each pair of no-slip and no-shear regions. Actually Philip's exact Stokes solutions are used as the corrections, separately for the longitudinal and transverse velocity component. Simulations were performed at $Re_\tau = 180$ in a standard $4\pi \times 2\pi$ periodic box, for a striped no-slip, no-shear surface with fill ratio 50% and a range of values of s^+. A sample result is given in figure 1.

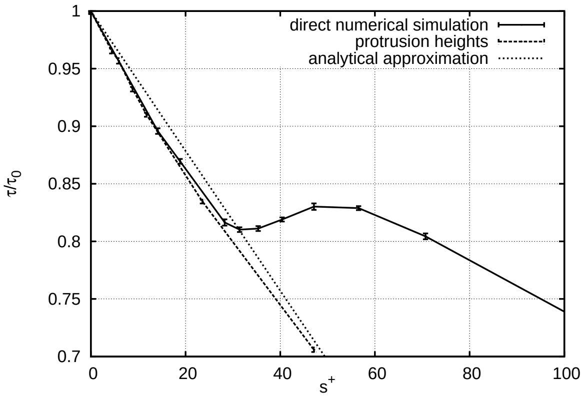

Figure 1: Drag reduction of a longitudinally striped no-slip, no-shear surface as a function of spacing in wall units.

As can be seen from this figure, though drag continues to decrease indefinitely with s^+ in contrast with what occurs for riblets, there is a distinctive kink between $s^+ \simeq 30$ and $s^+ \simeq 50$ separating regions with different behaviour. The dashed line in the same figure was obtained by repeating the turbulent simulation with a uniform surface as the wall on which slip boundary conditions are assigned, with the longitudinal and transverse velocity boundary condition each given its theoretical protrusion height. The comparison clearly shows that before the first kink ($s^+ \leq 30$), the action of the striped surface is really confined to Stokes flow and well described by the protrusion heights.

The same figure also contains a dotted straight line, which is simply the application of (1) with the theoretically calculated protrusion-height difference as obtained from Philip's solution. The agreement goes beyond what was ever confirmed for riblets, despite the crudeness of the underlying assumption.

Additional simulations with the surface striped across rather than along the flow direction were performed, and will be shown at the conference. These are important because a longitudinally striped air-laden surface, though being the most suitable for drag reduction, cannot resist the longitudinal pressure gradient tending to sweep the air bubbles away.

CONCLUSION

Superhydrophobic surfaces trap a discontinuous air layer through their texture which, in addition to changing the apparent contact angle of water drops, also changes the friction coefficient of a continuous water flow. Locally this effect can be represented through a slip coefficient [4], or equivalently through a protrusion height. More accurately, just as originally done for riblets [6], two different protrusion heights must be introduced, one for the longitudinal and one for the transverse velocity, and their difference is the only relevant physical parameter. Then, as long as the size (spacing) of the air bubbles confines their action to the Stokes region, the old result (1) can be used again to calculate the drag reduction. Turbulent numerical simulations, made possible by the inclusion of a local analytical correction in the numerical method, show that striped surfaces with spacing smaller than $s^+ = 30$ follow this law very closely.

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