

## A MODEL FOR TURBULENT DRAG REDUCTION OVER LIQUID INFUSED MICRO-TEXTURED SURFACES

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### INTRODUCTION

Superhydrophobic surfaces have been shown to reduce viscous drag in both laminar and turbulent flows [1, 2]. When submerged, air pockets can become trapped in the hydrophobic, microscope roughness elements, resulting in a heterogeneous surface composed of both air-water and solid-water interfaces. The presence of the air-water interfaces generates a localized slip effects, which, when aggregated across the entire surface, manifests itself as an effective slip length and can be associated with a significant reduction in skin friction drag when exposed to an external flow. For turbulent flows, it has been theorized that the most significant drag reduction occurs when viscous length scale is on the order of, or smaller than, the superhydrophobic groove width [1]. While the drag reduction effect can be significant, traditionally, retention of the air intermediary has been an issue for the practicality of superhydrophobic surfaces.

Liquid-infused surfaces have been proposed as an alternative to conventional superhydrophobic surfaces and have been shown to reduce drag in laminar flows [4]. Similar to superhydrophobic surfaces, liquid infused surfaces utilize an intermediary, wetting lubricant that is allowed to recirculate within the microstructures to allow for a localized slip effect at the lubricant-water interfaces. Several models and simulations have been proposed to model how flow over cavities can reduce drag, though a common assumption is to consider the interface between the intermediary lubricant, or air in the case of superhydrophobic surfaces, and the external fluid to exhibit perfect slip. This assumption is equivalent to having an inviscid lubricating fluid, which is inappropriate when considering liquid infused surfaces, where the intermediary can have a viscosity on the order of the working fluid, or larger. Such analysis was conducted on shear driven flow in Stokes flow [3] and will be built upon to model the effect of cavities on turbulent flow. To derive a model under these considerations, it is necessary to synthesize knowledge from flow in the inner region of the turbulent boundary layer and shear drive cavity flow. To simplify the interactions between the two regions, the linear subregion of the turbulent boundary layer is considered to behave analogously to Couette flow at sufficiently large Reynolds number and the flow within the cavity is assumed to behave as a shear driven cavity flow in the Stokes regime. For this model, a series of streamwise grooves is considered.

### FLOW IN CAVITY

The following model offers a prediction for the expected

drag reduction from a given liquid infused surfaces based on the microstructure geometry and viscosity ratio between the working fluid and lubricant. Given the small scale of the microstructures and order of the viscosity within the cavity, it is reasonable to assume flow within the cavity can be described by the Stokes flow regime. It is known that recirculation within the cavity gives rise to a slip velocity at the interface between the lubricant and external fluid. It is assumed that the magnitude of the localized slip velocity will be proportional to the depth of the recirculation center in then cavity and the viscosity ratio between the fluids. For rectangular grooves, the depth of the vortex, normalized by the cavity width, is well defined analytically in the Stokes regime and can be used as a parameter in the model [3]. This model assumes a perfectly flat interface between the lubricant and external fluid, namely infinitely large surface tension (or equivalently zero Capillary number), such that all lubricant is retained and flow within the cavity is only affected by the viscous shear at the interface and independent of all other external flow features. If 2-D rectangular grooves with aspect ratios larger than unity (depth exceeds width) are considered under these conditions, the effect of the cavities is readily modeled approximately as a parabolic distribution of localized slip such that the local slip is a maximum at the center of the cavity and zero at the edges.

### FLOW IN THE LINEAR SUBLAYER

Through scaling analysis of the turbulent 2-D channel equations, it can be shown that the inner region of a turbulent boundary layer at a sufficiently high Reynolds number can be parameterized by the viscous length scale, the friction velocity  $u_\tau \equiv \sqrt{\tau_w/\rho}$ , where  $\nu$  is the external fluid viscosity,  $\tau_w$  is the viscous shear stress,  $\rho$  is the external fluid density and the viscous length scale,  $\eta \equiv \nu/u_\tau$ . It can be further shown that when considering a heterogeneous boundary condition of flow over cavities, in the limit of the Reynolds number being sufficiently large such that the viscous length scale is smaller than width of the cavity,  $b$ , the interaction between the linear subregions over the cavities and solid surfaces is no longer leading order. With this consideration, the net drag reduction effect can be considered as an area weighted average of the Couette flow over a wall and Couette flow over a slipping cavity. To leading order, the near wall region can be approximated as linear up to  $5\eta$  from the wall where the velocity is  $5u_\tau$ . The shear stress over the cavity,  $\tau(x)$  is then given by

$$\tau(x) = \nu \frac{5u_\tau}{5\eta + \delta(x)}, \quad (1)$$

where  $\delta(x)$  is the local slip length and varies parabolically over the cavity. Modeling the localized length length as

$$\delta(x) = bDN \left(1 - \frac{x^2}{b^2}\right) \quad (2)$$

with  $b$  as the cavity width,  $D$  as the depth of the cavity vortex normalized by  $b$  and the viscosity ratio between the external fluid and lubricant,  $N$ . Simply integrating (1) over a cavity period and letting  $\delta = 0$  over solid boundaries gives the area-weighted average shear stress and an estimate for the percentage drag reduction. Thus the percentage drag reduction,  $DR$  can be shown to be calculated with

$$DR = a - \frac{a}{DNb^+} \sqrt{\frac{25}{1 + \frac{5}{b^+DN}}} \operatorname{arctanh} \left( \sqrt{\frac{1}{1 + \frac{5}{DNb^+}}} \right), \quad (3)$$

where  $a$  is defined as the lubricant area fraction and  $b^+ \equiv b/\eta$ .

## RESULTS

Several interesting results from this equation become evident at first glance. In the limit of infinite Reynolds number, the percentage drag reduction should approach the lubricant area fraction irrespective of the viscosity ratio between the two fluids. While there is little data for drag reduction over liquid infused surfaces, because the model is dependent solely on the cavity geometry, viscosity ratio and flow Reynolds number, the model can be considered for superhydrophobic surfaces as well. Figures 1 and 2 show the drag reduction data for superhydrophobic streamwise grooves 30 and 60 microns wide, respectively.

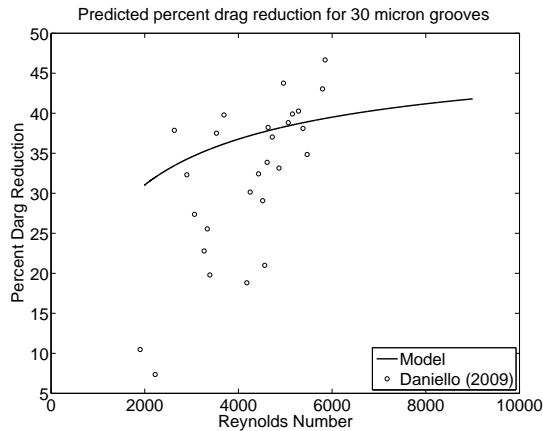


Figure 1: Comparison of model prediction with data from [1] for 30 micron streamwise grooves.

## CONCLUSIONS

While the further experimental data is needed to verify the model, agreement with current superhydrophobic data is promising. The model presented provides insight into a possible mechanism that could generate drag reduction in both superhydrophobic and liquid infused surfaces and links the localized slip length to the viscous length scale and the geometric length scale. Though the relevancy of this model is

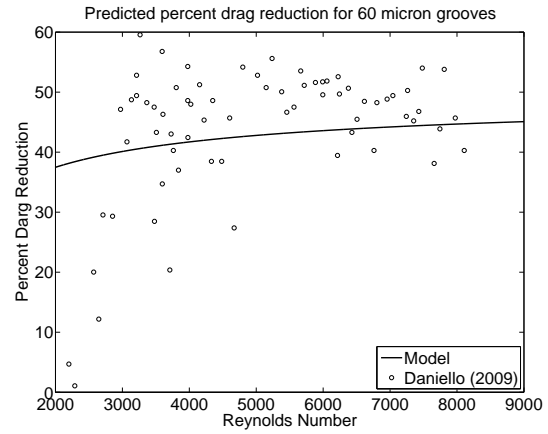


Figure 2: Comparison of model prediction with data from [1] for 60 micron streamwise grooves.

most appropriate for turbulent flows at high Reynolds number, where the linear sublayer is well defined, the insight to the interplay of the relevant length scales can perhaps provide insight into other flow regimes.

## ACKNOWLEDGMENTS

This work was supported under ONR MURI Grants N00014-12-1-0875 and N00014-12-1-0962 (Program Manager Dr. Ki-Han Kim).

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