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LINEARLY OPTIMAL SPANWISE BOUNDARY MODULATIONS FOR WAKE INSTABILITY CONTROL

O. Tammisola
Faculty of Engineering, University of Nottingham, NG7 2RD, Nottingham, UK
and
KTH Mechanics, KTH Royal Institute of Technology, 10044, Stockholm, Sweden

INTRODUCTION

This paper addresses the optimization of spanwise-modulated suction, and boundary shapes such as riblets, for control of the instability of bluff-body wakes. A successful suppression of wake instability would lead to a suppression of lift/drag oscillations and an alteration of turbulence level and structure at higher Reynolds numbers.

Control design for linear modal instability is usually performed by first-order adjoint-based sensitivity analysis [6], by taking the first derivative of the instability growth rate with respect the location of a passive or active device, *e.g.* a small control cylinder or wall suction. Similar gradient-based techniques exist in the industry for obtaining gradients used in shape optimization to minimize drag in steady flows [4].

The first-order sensitivity analysis as above predicts zero net effect of spanwise-periodic modulations such as a spanwise-alternating suction, or riblets, on wake instability. On the contrary, experimental and numerical findings show that spanwise-sinusoidal steady suction/blowing stabilizes wake instability very efficiently [5, 3]. Recently, this stabilizing effect has been explained and reproduced by considering the second derivative of the instability growth rate with respect to the base flow modifications induced by spanwise-sinusoidal suction of different wavelengths and comparing to a second-order "wavemaker" position [7]. The optimal base flow modifications can also be constructed by forming the whole second-order (Hessian) matrix, and computing its extremal eigenvalues and eigenvectors. This step however can become computationally extremely expensive. Very recently, the Hessian was explicitly computed in a 1D problem [1], and optimal base flow modifications found. However, the method involved explicit matrix inversion, preventing an immediate generalization to 2D and 3D.

In this paper, we will review some of the recent progress on wake instability control by spanwise-modulated suction. In this part, we will focus on the wake behind a flat plate. As a novel development, we will introduce an approach to compute the optimal spanwise-modulated shapes and boundary suction profiles, which is applicable to 2D and 3D flows. The goal is to obtain the most stabilizing spanwise-sinusoidal undulation shape for the 2D wake behind a cylinder, and from this construct optimal "riblet" shapes for the same (the "optimal riblet" here refers to an optimal spanwise-periodic shape change, whether sharp or smooth). Here, we utilize the natural parameterization of the surface to construct a reduced-order Hessian, which should make the technique suitable for large and 3D problems.

PERTURBATION ANALYSIS

The wake instability is a global linear instability problem of the general form:

$$\mathcal{L}\mathbf{q}_0 = \sigma_0 \mathbf{q}_0, \tag{1}$$

where $\mathcal{L}$ is the linear Navier-Stokes operator, $\mathbf{q}_0$ is an eigenvector, and the eigenvalue $\sigma_0 = \sigma_r + i\sigma_i$ gives the growth rate (σ_r) and frequency (σ_i) of the instability mode, and $\mathbf{q}_0$ gives the mode shape. Now let us introduce spanwise-sinusoidal undulations of unknown shape:

$$\delta h(s,z) = \delta\tilde{h}_+(s)\exp(i\beta_B z) + \delta\tilde{h}_-(s)\exp(-i\beta_B z) \tag{2}$$

To obtain changes in σ_0 induced by this undulation, let us consider a perturbation series around the spanwise-constant solution:

$$(\mathcal{L} + \epsilon\delta\mathcal{L})\left\{\mathbf{q}_0 + \epsilon\mathbf{q}_1 + \epsilon^2\mathbf{q}_2 + O(\epsilon^3)\right\} = \left(\sigma_0 + \epsilon\sigma_1 + \epsilon^2\sigma_2 + O(\epsilon^3)\right)\left\{\mathbf{q}_0 + \epsilon\mathbf{q}_1 + \epsilon^2\mathbf{q}_2 + O(\epsilon^3)\right\} \tag{3}$$

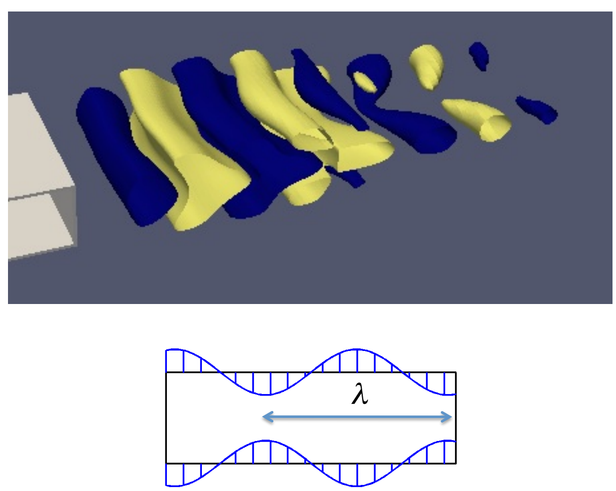

Figure 1: Spatial structure of the global eigenmode with spanwise-alternating suction of the most stabilizing wavelength $\lambda = 1.1$. From [7].

where the operator $\delta\mathcal{L}(\delta h)$ can be computed, and introduces a perturbation of the eigenvalue due to undulations. By grouping together terms of any given power of ϵ, we can generate approximations of the change in instability growth rate accurate up to that order. For spanwise wavy base flow modulations as the ones induced by riblets, σ_1 is known to equal zero [3, 2]. Hence, the second-order eigenvalue drift σ_2 is needed to determine the stabilization/destabilization.

We will derive and compute the second-order eigenvalue drift with respect to spanwise-wavy suction and spanwise-wavy shapes. Preliminary results indicate that the spanwise-periodic surface modifications may be just as efficient as spanwise-alternating boundary suction in suppressing wake instability.

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